

**PRECISION TESTS OF GENERAL RELATIVITY AND OF THE EQUIVALENCE PRINCIPLE
USING ULTRASTABLE OPTICAL CLOCKS: A MISSION PROPOSAL**

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ABSTRACT

Within the framework Cosmic Vision 2015-2025, and as part of the Fundamental Physics Theme "Exploring the limits of contemporary physics", we propose to carry out a space mission to test with high precision aspects of the Equivalence Principle (Local Position Invariance, Local Lorentz Invariance) and a number of Predictions of General Relativity (Gravitational Redshift, Post-Newtonian orbital effects). The science goals include the most precise test of a General Relativity prediction, with a relative accuracy of 10^{-9} . The significant improvement in accuracy of the tests will be possible by using ultrastable optical clocks and ultraprecise time transfer, combined with drag-free satellite technology.

Key words: Local Position Invariance, Gravitational Redshift, Atomic clocks, Molecular clocks, Equivalence Principle, Lense-Thirring Effect, Perigee shift, Local Lorentz Invariance, Isotropy of Space, Laser, Fine-structure constant, Masses of nuclei, Mass of electron, Laser cooling

indicate the path toward a deeper understanding of the structure of nature.

The space mission proposed here is based on the OP-TIS mission concept [2; 3; 4], and is related to the SPACE-TIME mission concept [5]. It will probe two aspects of the Equivalence Principle, LPI and LLI. A second group of tests is devoted to consequences of General Relativity: the absolute gravitational redshift and non-Newtonian effects on the orbit of the spacecraft.

The Principle of Local Position Invariance states that the outcome of any local experiment is independent of where in the universe it is performed. When applied to the internal oscillations of atoms or molecules ("clocks"), this principle implies that the ratio of transition frequencies of co-located quantum systems is a universal quantity, and does not depend on e.g. how close the systems are to a large mass, such as a star or a planet. This manifestation is also denoted by universality of gravitational redshift. Terrestrial experiments have verified this universality up to the 10^{-5} -level. As consequence of LPI and assuming the validity of the Standard Model, the values of the fundamental constants determining the transition frequency ratios (e.g. α , m_e/m_p , and other mass ratios) are not affected by the presence of mass and its gravity.

The absolute gravitational redshift has been measured by a number of experiments, the most accurate being the GP-A experiment, that achieved a relative precision of $7 \cdot 10^{-5}$ using a Hydrogen maser [6]. One of the objectives of the ESA ACES/PHARAO mission to be flown on the ISS in 2009 is to bring this uncertainty down to $2 \cdot 10^{-6}$.

In this paper, we concentrate on a central aspect of the mission payload, the optical clocks. We will point out that the recent and ongoing developments in quantum optics are leading to a new generation of clocks with instabilities orders of magnitude lower than that of the current state-

1. INTRODUCTION

The Principle of Equivalence is a foundation of today's accepted theory of gravitation, General Relativity [1]. It contains the Weak Equivalence Principle, the Principle of Local Position Invariance (LPI), and the Principle of Local Lorentz Invariance (LLI). The latter is also a fundamental assumption of the Standard Model.

As there is no quantum theory of gravitation, experiments are needed to explore the limits of today's theories, in the hope of discovering tiny deviations that may

of-the-art clocks. This enabling technology will allow to reach extremely high accuracy in the proposed mission.

2. SCIENCE GOALS

The science goals are as follows:

1. A test of LPI by testing the universality of the gravitational redshift for an ensemble of atomic and molecular optical clocks. Relative accuracy $1 \cdot 10^{-10}$, 5 orders improvement compared to the best tests so far. This allows the first test of the universality at a level of second order in $\Delta U/c^2$.
2. A measurement of the absolute gravitational redshift, by comparing the frequency of the satellite clocks with a reference clock. Relative accuracy: $1 \cdot 10^{-9}$, four orders better than present tests.
3. A test of LLI for photons and electrons by comparing the frequencies of a set of optical cavities oriented at different angles. Relative accuracy: $1 \cdot 10^{-20}$, 4 orders better than terrestrial tests.
4. A test of LLI for photons and electrons by comparing the vibrational frequencies of molecules oriented in different directions in space, relative accuracy 10^{-19} (no terrestrial tests so far).
5. A test of LLI for neutrons and protons by comparing the transition frequencies of atomic Zeeman levels.
6. A measurement of the Lense-Thirring effect (two orders improvement)
7. A measurement of perigee advance. Relative accuracy: $3 \cdot 10^{-4}$ (one order improvement)
8. A test of the $1/r$ -Newtonian potential on the distance scale corresponding to the orbit size, at the 10^{-12} -level (one order improvement).

3. KEY SATELLITE AND ORBIT FEATURES

Key parameters of the satellite's orbit are a high eccentricity and a high apogee (for 1,2). Drag-free operation is required for goals 6,7,8. Mission duration is several years.

The satellite must spin around its axis at a very stable rate (for 3,4,5). Laser ranging is performed for accurate orbit determination (for 2,6,7,8), and this implies the use of an Earth-orbiting satellite.

An eccentric orbit around a large, dense planet would lead to a favourably large gravitational potential variation. The maximum variation is on the order of the potential at the planet's surface (radius R). For Jupiter, $U(r = R)/c^2 = 2 \cdot 10^{-8}$, while for Earth the value is $7 \cdot 10^{-10}$. This implies that the frequency shift contribution of second order in U is about a factor 1000 larger for a Jupiter-orbiting satellite as compared to Earth. This advantage would be fully usable for the gravitational frequency shift universality test. However, the measurement of the absolute redshift would be more complex for a satellite orbiting Jupiter, since a long-distance time link to Earth would be required, probably with lower performance

compared with a time link to an earth-orbiting satellite. Here, a second satellite orbiting Jupiter with a reference clock could be an improved scenario.

4. CLOCKS

Current best microwave clocks (Cs) have stabilities of $1.6 \cdot 10^{-14}$ at one second and the comparison between two independent fountains reaches $2.2 \cdot 10^{-16}$ at 50 000 seconds [9]. The projected stability of the ACES/ PHARAO cold atom microwave clock in space is $1 \cdot 10^{-13}$ at one second or $4 \cdot 10^{-16}$ at 50 000 seconds. It is believed that the accuracy of these devices, currently $6 \cdot 10^{-16}$, will not improve by more than an order of magnitude.

Optical clocks will thus be the key technology to be used on the proposed mission. First-generation optical atomic clocks demonstrated at laboratories such as PTB and NIST have already shown to surpass the best microwave clocks in stability on short timescales (tens of minutes to hours). For example, relative instabilities of $\sim 3 \cdot 10^{-16}$ at 1000s have recently been achieved with single-ion clocks as well as with a clock based on ballistically expanding cold Ca atoms [7,8]. This is about a factor 5 lower than the best results for microwave fountain clocks (Cs). Absolute uncertainties of $\sim 3 \cdot 10^{-15}$ have already been achieved with single-ion clocks [10].

By implementing optical clocks with a large number of particles ($\sim 10^7$) trapped in the Lamb-Dicke regime in an optical lattice [11] or in an electrodynamic trap, an instability at the level $1 \cdot 10^{-18}$ on a timescale of several hours appears realistic. The quantum limit could be as low as 10^{-20} for such particle numbers. Assuming a set of dissimilar clocks with 10^{-18} -instability over the timescale of one orbit, and the possibility of averaging over ~ 1000 orbits, a test of LPI at the level stated above will be feasible.

A good choice of atom species and transitions is such that the frequency ratio ω_1/ω_2 of a pair of clock transitions depends strongly on the fine-structure constant α , i.e. exhibits a large value of $A = d \ln(\omega_1/\omega_2)/d \ln \alpha$. A hypothetical LPI-violating coupling $\alpha(U)$ to the gravitational potential U would then lead to a relatively large effect. Two examples are:

- (i) Sr and Yb on the $^1S_0 \rightarrow ^3P_0$ transitions (two clocks currently under development at NIST, PTB, SYRTE, Firenze U., Tokio U., Düsseldorf U.), with a derivative $A = -0.25$ [12];
- (ii) a single atomic species, where a comparison between two appropriate electronic transitions is performed: in the Yb⁺-ion, the two clock transitions already being explored, at 436 nm (PTB [13]) and 467 nm (NPL [14]), have a derivative of $A = 6.1$ [15].

Molecular optical clocks are complementary to atomic clocks. The vibrational frequencies in molecules depend on the masses of the involved particles, i.e. of the electron and of the nuclei. Therefore, the frequencies depend also on the strength of the strong interaction force. Ratios of vibrational transition frequencies within the same molecule can

depend strongly on the mass ratios, if one transition involves a highly excited vibrational state [16]. Hence molecular clocks can probe the coupling of the strong force to the gravitational potential. The use of clocks whose transition frequencies depend on the electron mass is also highly desirable from the point of view of string theory models [17], which predict a much larger time variation of m_e/m_p compared to α [18].

The technology of preparing ultracold molecules has recently been developed both for charged and neutral molecules [19; 20; 21; 22]. It is expected that optical clocks based on vibrational transitions in molecules will reach similar instabilities as atomic optical clocks. Two possibilities are clocks based on single molecular ions at rest in an electrodynamic trap or on neutral molecules trapped in an optical lattice.

Apart from clocks whose frequencies depend on α and the masses of particles, clocks involving the weak interaction are desirable in order to complete the set of forces to be tested. The parity-violation induced frequency splitting between certain vibrational levels of mirror molecules could in principle be used [23]. Although such a splitting has not been observed so far due to its smallness, the developments in ultracold molecules spectroscopy may make this possibility realistic in the near future.

Optical frequency comparisons between the various clocks will be performed using femtosecond frequency combs, which are already available as compact, diode-pumped systems.

It should be pointed out that the concept of quantum state entanglement, already experimentally demonstrated for up to $N = 6$ ions so far, could be applied to clocks. If successfully implemented on large ensembles, a further significant improvement ($\sim \sqrt{N}$) in stability could be obtained.

For the implementation of optical clocks, lasers with linewidths well below 1 Hz will be required. This is accomplished using frequency stabilization to low-loss optical cavities [24; 25; 26]. The required systems can also be used for a test of the constancy of the speed of light (LLI for the photon) [27; 28], by orienting two cavities at right angle and spinning the satellite. The advantage of a satellite experiment compared to a terrestrial experiment [28] is the absence of gravity and thus of gravitationally induced cavity deformations (the effects of the gravity gradient can be kept sufficiently small [3]). By using cavities made of different materials, LLI is simultaneously tested with respect to electron motion and photon propagation [29].

Techniques similar to those used by the optical clocks will also be used for the tests of Lorentz invariance of particle spin properties and isotropy of the Coulomb potential (goals 4 and 5 above) [30].

Time transfer techniques between the satellite clocks and Earth clocks (or a master clock on a satellite with dissimilar orbit, e.g. on the ISS) with accuracy comparable

to the instability of the clocks will have to be developed [31]. Current time transfer techniques have a noise level of 50-100 picoseconds per day, currently limiting clock comparison at the 10^{-15} level over several days of averaging. An important step forward will be the ACES microwave link with a projected 5ps/day noise level, enabling space to ground optical clock comparisons at a level of $5 \cdot 10^{-17}$ /day as well as ground to ground clock comparisons at a level of few 10^{-17} over several days.

5. CONCLUSION

Rapid developments in quantum technology are leading to a new generation of clocks based on electronic transitions in appropriately prepared cold atom samples, with an instability several orders of magnitude lower than that of fountain clocks. In addition, molecular clocks of similar stability are likely to soon emerge, widening substantially the scope of a clock mission. A future mission flying clocks on a dedicated satellite with highly eccentric orbit represents a very powerful probe of the Principle of Local Position Invariance and of the gravitational frequency shift (an effect sensitive to the time component g_{00} of the metric tensor). Such a mission is the natural extension of the type of mission to be pioneered with the ACES mission. It will be complementary to MICROSCOPE and STEP (tests of the Weak Equivalence Principle), as well as LATOR (test of light deflection, an effect also sensitive to the spatial components of the metric tensor). Taken together, these missions will deepen our knowledge of the basic principles of gravity by many orders of magnitude.

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