Repeated Quantum Nondemolition Measurements of Continuous Optical Waves

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Repeated quantum nondemolition (QND) measurements for continuous optical waves were performed by combining a degenerate optical parametric amplifier and a squeezed-light beam splitter. Two-step quantum state preparation and three-beam entanglement of the individually squeezed output beams were demonstrated. The experiment is analyzed using a set of generalized criteria for nonideal individual and repeated QND measurements. The system was operated fully stabilized for up to 36 hours.

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A QND measurement (QNDM) seeks to measure a QND observable precisely and without perturbation. The future evolution of the QND observable can be predicted precisely [1]. Examples of QND observables are the energy and the quadrature operators of a harmonic oscillator, a counterexample being the position of a free particle. QNDM’s are indirect measurements. The quantum state under investigation (signal) is first coupled to a second quantum state (meter). For appropriate coupling, the pertinent signal QND observable is not disturbed during the interaction and can be measured by a subsequent measurement of a meter observable. QNDM strategies are important since they allow one to circumvent quantum mechanical limits inherent to standard measurement techniques.

The feasibility of QNDM’s was established by a series of experiments on optical waves, using $\chi^{(3)}$ couplings in optical fibers [2], atomic beams and cold atoms [3], and $\chi^{(2)}$ couplings in crystals [4,5]. A long-standing challenge in the field has been the demonstration of repeated QNDM’s, which represents the purpose of the QND concept. The first demonstration was recently given by Bencheikh et al. [6] using two pulsed phase-sensitive optical parametric amplifiers (OPA’s) to measure the amplitude quadrature of a pulsed wave.

In this Letter we present (i) the first demonstration of a central feature of (nonideal) repeated QNDM’s, two-step quantum state preparation; (ii) an in-depth characterization using an extended set of criteria for nonideal QNDM’s; (iii) operation in the continuous-wave (cw) regime; and (iv) reliable, long-term stable operation. The two latter features are of importance for future application of QND techniques to precision experiments.

The basic layout of a repeated QND scheme for a quadrature operator of an electromagnetic field mode (“beam”) is shown in Fig. 1. When ideal QNDM’s of the same observable of a signal input state $S_0$ are repeated [7], the first and the subsequent measurements perform different operations. In general, the signal input state $S_0$ is not an eigenstate of the measured observable, and the first measurement will cause a state reduction and necessarily perturb the state. Its output state $S_1$ will be a specific eigenstate, whose eigenvalue can be identified by the meter output $M_1$. Once these known eigenstates are prepared, it is possible to demonstrate that a following QND system leaves any eigenstate unchanged and provides its eigenvalue. Thus, system 2 clearly realizes the axiom of quantum measurement theory for the QND observable.

In practice, ideal QNDM’s are impossible to realize due to finite coupling strength of the signal-meter interaction and unavoidable propagation inefficiencies. Therefore, criteria for individual QNDM’s have been developed [8] in order to distinguish between standard destructive quantum measurements and those that follow the QND strategy successfully. In the following, we extend the criteria for individual QNDM’s, to signal inputs $S_0$ with total (quantum and classical) noise of the observable differing from the coherent-state level.

1. Quantum optical tap (QOT) condition.—A QOT was defined to satisfy $T_{S_1S_0} + T_{M_1S_0} > 1$, where the transfer coefficient $T_{out,in}$ is the fraction of the signal-to-noise ratio (SNR) of a classical modulation of the input observable that is transferred to the output observable. However, this criterion can be fulfilled by a simple beam splitter, if $S_0$ carries excess noise. We propose a new criterion $T_{S_1S_0} + T_{M_1S_0} > 2S_0/1 + S_0$, which rules out any phase insensitive device that decouples amplitude and phase quadratures [9]. $S_A$ denotes the quantum noise (“squeezing”) of the observable of state $A$ relative to the coherent state level.

2. Quantum state preparation (QSP) condition.—The conditional variance $V_{S_1|M_1} = S_{S_1}(1 - C_{S_1M_1}^2)$ is the remaining uncertainty of the observable of $S_1$ after a measurement of $M_1$. $C_{A,B}$ denotes the correlation coefficient between the observables of the states $A$ and $B$.

FIG. 1. Schematic for repeated QND measurements.
operational interpretation of the conditional variance is that the information contained in $M_1$ can be used to reduce the uncertainty (noise) of $S_1$ to the level $V_{S_1|M_1}$ using an appropriately driven modulator. QSP was defined by $V_{S_1|M_1} < 1$, the ability to generate a squeezed state. However, by instead mixing the quadratures $S_1$ and $M_1$ at an appropriate beam splitter, an output with a minimum signal. This is described by the following identities:

$$S_{\text{min}} = \frac{S_{S_1} + S_{M_1}}{2} - \sqrt{\left(\frac{S_{S_1} - S_{M_1}}{2}\right)^2 + S_{S_1}S_{M_1}C_{S_{1,M_1}}^2}$$  (1)

can be prepared. As $S_{\text{min}}$ is (up to 3 dB) lower than $V_{S_1|M_1}$ [9], the criterion $S_{\text{min}} < 1$ for QSP is thus more general than $V_{S_1|M_1} < 1$, and extends the set of quantum state preparers.

3. Entanglement.—If $S_0$ carries technical noise, the observables of $S_1$ and $M_1$ may be correlated without being entangled. Entanglement requires a nonzero correlation coefficient $C_{S_1,M_1}^Q > 0$ between the quantum fluctuations of the respective observables. This can be tested experimentally by the criterion

$$\frac{|C_{S_1,M_1}| - \sqrt{T_{S_1,S_0}T_{M_1,S_0} \epsilon}}{(1 - T_{S_1,S_0})(1 - T_{M_1,S_0} \epsilon)} > 0,$$  (2)

where $\epsilon = 1 - S_{S_0}^{-1}$, since the left-hand side is a lower bound for $C_{S_1,M_1}$ if the system decouples amplitude from phase quadratures, and if $S_0$ is the only input with noise.

$$C_{S_2,\text{opt}(M_1,M_2)}^2 = (C_{S_2,M_1}^2 + C_{S_2,M_2}^2 - 2C_{M_1,M_2}C_{S_2,M_1}C_{S_2,M_2})/(1 - C_{M_1,M_2}^2).$$  (3)

E. $T_{\text{tot},S_0} = T_{M_1,S_0} + T_{M_2,S_0} + T_{S_2,S_0} > L$.—A specific lower bound $L = 3S_{S_0}/(2 + S_{S_0})$ is suggested as this is the maximum achievable with two vacuum-meter beam splitters [11].

If the fluctuations in both QND systems propagate linearly and all input fluctuations are independent, then a quantum correlation is propagated as described by the respective transfer coefficient, exactly like a classical signal. This is described by the following identities:

$$|C_{S_2,M_1}| = |C_{S_2,S_1}|C_{S_1,M_1}^2 - T_{S_2,S_1}|C_{S_1,M_1}|,$$  (4)

$$|C_{S_2,M_2}| = |C_{S_2,S_1}|C_{S_2,M_1}^2 - T_{S_2,M_2}|C_{S_1,M_1}|.$$  (5)

F. A necessary condition implied by (4), (5), that is directly accessible during a repeated QNDM, is $U = (|C_{S_2,M_1}|/|C_{S_2,M_2}|)(T_{M_2,S_0}/T_{S_2,S_0}) = 1$.

If—complementary to performing the repeated QNDM's—the observable of $S_1$ is directly accessed, a more complete characterization of the system using two additional criteria becomes possible.

G. QND criteria 1–3 are fulfilled for the individual QND systems. The relations $T_{S_2,S_0} = T_{S_2,S_1}T_{S_1,S_0}$ and not at the coherent-state level. For low classical input noise ($\epsilon = 0$) $C_{S_1,M_1}^Q \approx C_{S_1,M_1}$. Results

A further important parameter of a QND system is the gain of the signal channel, for which, however, no criterion can be given. We emphasize that nonunity gain is compatible with predictability of the signal output, and that applications for QND systems exist which do not require unity gain.

Complementary to individual characterization, repeated QNDM's also must satisfy certain criteria. Evidently, satisfaction of the above criteria 1–3 for the two individual measurements systems is one requirement to be placed. They cannot, however, be tested while repeated QNDM's are performed, since access to $S_1$ is needed.

A. $C_{M_1,M_2}^Q > 0$.—Both measurements return correlated quantum information, which Yurke [10] coined “the needles move together.”

B. $C_{S_1,S_2}^Q > 0$.—This ensures that the quantum state that is prepared by QND 1 is measured at least partially nondestructively by QND 2.

C. $C_{M_2,S_2}^Q > 0$.—QND 2 is able to generate entangled outputs. Criteria A–C assert that all three output states are entangled.

D. Two-step QSP.—If all three output states are entangled, both nonideal measurements of $M_1$ and $M_2$ lead to a partial state reduction of the full state, i.e., a two-step QSP of $S_2$. This is demonstrated when the observable of $S_2$ can be predicted more precisely by using both $M_1$ and $M_2$ readouts than when using only one readout. The system must satisfy $C_{S_2,\text{opt}(M_1,M_2)}^2 \geq \max(C_{S_2,M_1},C_{S_2,M_2})$. The predicted optimized correlation is

$$T_{M_2,S_0} = T_{M_2,S_1}T_{S_1,S_0},$$

which follow from the definition of the transfer coefficients, can be applied to predict the signal transfer properties of the two-step measurement.

H. Relations (4) and (5) hold individually, not only $U = 1$.

Experiment.—We have combined two different systems, which were previously shown to individually satisfy criteria 1–3, to perform repeated QNDM’s: a degenerate type-I optical parametric amplifier and a squeezed light beam splitter [5,12].

The experimental setup for the combined system is shown in Fig. 2. A continuous-wave Nd:YAG laser (500 mW, 1064 nm) is the primary laser source. Most of its output power is frequency doubled to provide 241 mW at 532 nm, which is split to pump the OPA in QND 1 as well as the squeezer (QND 2). Bright amplitude squeezed light (3.8 dB, 30 µW) is produced by an injection-seeded, semi-monolithic LiNbO$_3$ OPA [13]. A small fraction of the laser output is amplitude modulated at 13.3 MHz and provides a signal input SNR of 38.8 dB. QND 1 is a monolithic dual-port LiNbO$_3$ ring resonator [5] exhibiting a nonlinearity $\Gamma = 1.0$ kW$^{-1}$ and a free
spectral range of 10.2 GHz. The signal beam is resonantly injected into the crystal through its dielectric coated front face (transmission: 1.44%, mode match: 90.6%). A piezo mounted prism acts as a variable transmission coupler for the resonating wave to the meter 1 beam via frustrated total internal reflection. The reflected signal beam propagates to a dielectric 50/50 beam splitter (QND 2, transmission: 49.7%), where it is coupled to the squeezed meter 2 input beam (mode match 80%). Servo controls are used to stabilize resonances, pump phases, and the temperatures of various components.

The amplitude quadrature fluctuations (including modulations) of all beams are analyzed at balanced self-homodyne detectors HD1–4, which employ InGaAs photodiodes with a quantum efficiency of 97%. Optical propagation efficiencies are 99.4% from HD 1 to QND 1, 98% from QND 1 to QND 2 and 99.2%, 99.2%, and 99.3% for HD 2–4.

For a careful characterization of the properties of the combined system, we performed measurements of the transfer coefficients from the signal input beam to all output beams, of the correlation between the output beams and of the noise levels of the input and output beams. The transfer coefficients are determined by the ratio of the photocurrents of the output beams relative to that in the signal input beam. The SNR’s are evaluated from the ac signals of the HD’s by comparing the modulation strength at the modulation frequency with the amplitude noise at a frequency nearby (14.2 MHz). The correlation coefficients are obtained by interfering the currents of two output beams with correct phase and variable attenuations [5].

Turning now to the results, Fig. 3 shows the three individual and as well as the total transfer coefficients as a function of the ring resonator outcoupling $T_2$. The total transfer coefficient exceeds the lower bound given by criterion $E$ (1.03 in our case), over a wide range of outcoupling transmissions, with a maximum of 1.18. Results of correlation measurements are presented in Fig. 4. The theoretical predictions use only measured parameters except for the internal losses of the pumped ring resonator, which are fitted to 0.99% [5].

The QND properties of the system are optimized at $T_2 = 6.2\%$. Evaluating the 20 nearest measurements, we obtain for the noise levels: $S_{10} = 0.24 \pm 0.14$ dB (small excess input noise), $S_{1M} = -2.67 \pm 0.07$ dB, $S_{2M} = -1.01 \pm 0.05$ dB, $S_{2S} = -0.76 \pm 0.07$ dB. The transfer coefficients are $T_{1M,10} = 0.49 \pm 0.01$, $T_{1M,20} = 0.336 \pm 0.006$, $T_{2S,20} = 0.339 \pm 0.007$, $T_{\text{tot},10} = 1.17 \pm 0.02$ (criterion $E$), and the correlations are $C_{1M,12} = 0.24 \pm 0.05$ (criterion $A$), $C_{1M,2S} = 0.23 \pm 0.03$ (criterion $B$), $C_{2M,2S} = 0.16 \pm 0.03$ (criterion $C$). The deviations given are the statistical errors of a single measurement. From these measurements we can also verify consistency of our data with criterion $F$, obtaining $U = 0.93 \pm 0.28$. There are altogether six conditional variances for the system; the lowest one is $V_{1M|2} = -2.93 \pm 0.13$ dB.

To test the two-step QSP ability experimentally, an appropriate linear combination of both meter output photocurrents is superimposed with the signal output photocurrent; see inset in Fig. 5. The optimized correlation obtained is the minimum total noise achievable as a function of $g, g_2$. For $T_2$ stabilized to 7.5%, an optimized correlation $C_{S2,M1|T_2,\text{opt}} = 0.250 \pm 0.004$ was measured, exceeding the individual correlations $C_{S2,M1} = 0.218 \pm 0.003$ and $C_{S2,M2} = 0.190 \pm 0.002$, respectively.
In completely stabilized operation repeated QND measurements could be performed over prolonged periods without manual intervention. Figure 6 shows the longest measurement extending over a period of 36 h.

In conclusion, we have implemented repeated QNDM’s in the cw regime for the first time, and demonstrated excellent long-term operation, showing that QND techniques have potential for applications. The system agrees with criteria $A - F$, which enable verification of repeated QNDM operation. In particular, we have explicitly demonstrated two-step quantum state preparation. Furthermore, the performance of the system is in agreement with theoretical calculations that take into account various measured inefficiencies. Finally, the three output waves generated by the repeated QNDM system represent individually squeezed triple beams.

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[11] This limit is exceeded by a high-gain nondegenerate optical parametric amplifier (OPA) followed by a beam splitter: $T_{0.5,0.5} = 1.5$ even if $S_{0.5} = 1$. Though this amplifier is generally considered phase insensitive, it satisfies the QSP criterion and is therefore a nonclassical device.