Quantum Statistics of the Squeezed Vacuum by Measurement of the Density Matrix in the Number State Representation

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We report the reconstruction of the quantum state of squeezed vacuum generated by a continuous-wave optical parametric amplifier. Homodyne detection and tomographic reconstruction methods were used to obtain the density matrix in the Fock (number state) representation. The photon number distribution exhibits odd-even oscillations, a manifestation of the photon pair production in the second-order nonlinear medium. [S0031-9007(96)01320-8]

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A single-spatial mode of the electromagnetic field in the optical frequency domain represents a particularly clean and accessible example of a quantum harmonic oscillator system. Quantum statistical effects arising from the noncommuting quadrature components of the electric field of light waves have been clearly established as a result of extensive experimental studies. Ultimately, a complete characterization of a quantum state implies the determination of its wave function (for a pure state) or, more generally, of its density matrix. Performing such studies is of importance because they represent a clear application of the principles of quantum measurement. On a practical side, optical quantum state analyzers will likely become useful tools in the future, for example, to characterize particular quantum states of the light field that may be injected into optical instruments such as interferometers or polarimeters so as to improve significantly their sensitivity [1].

Historically, the characterization of optical quantum states was first pursued using photon counting and was applied to study the statistics of thermal [2], coherent [2], sub- and super-Poissonian [3], and quantum correlated light fields [4]. This technique is particularly useful for determining the second-order correlation function, where detector quantum efficiency is not a limiting factor. More detailed information, the photon number probability distribution $p(n)$, i.e., the diagonal elements $\langle n|\rho|n\rangle$ of the density matrix, where $|n\rangle$ is the n-photon Fock state, may in principle be obtained by recording with an ideal photon counter the statistics of photon emission events from the light source. Accurate measurements of $p(n)$ of nonclassical states require counters with appropriately high temporal resolution, near-unity quantum efficiency, and single photon resolution. As photon counters with this combination of properties are not yet available, such measurements are, however, not possible at present. A second common method, which has been widely applied to the characterization of squeezed light [5,6], measures the variances of the electric field quadratures (via photocurrent spectral densities) using balanced homodyne detection [7] with high efficiency photodetectors lacking single photon sensitivity.

A full characterization of optical quantum states is possible beyond the capabilities of the above approaches. As shown by Bertrand and Bertrand and Vogel and Risken [8], homodyne detection of the state followed by tomographic reconstruction permits one to recover the Wigner quasiprobability distribution function from experimental data. The density matrix in position or momentum representation is then easily obtained by Fourier transformation. The technique is a precise one thanks to the availability of near-unity quantum efficiency (high-flux) photodetectors. Smithey *et al.* [9] performed the first experiment of quantum state tomography and reconstructed the Wigner function of pulsed squeezed light using the inverse Radon transform.

Here we report the reconstruction of the density matrix in the number state basis of a strongly nonclassical state of the light field. The squeezed vacuum state generated by a continuous-wave optical parametric amplifier (OPA) [6,10,11] is measured and reconstructed using elegant algorithms recently developed [12–14], which yield all density matrix elements without passing through the intermediate step of reconstructing the Wigner function. The experimental verification of oscillation in the photon number distribution is made possible by using an OPA that emits strongly (5.5 dB) squeezed vacuum and a highly efficient homodyne detector.

The principle of the experiment is sketched in Fig. 1. A quantum state generator, here a parametric oscillator pumped below its threshold by a continuous-wave laser, emits a light field on which a large number of measurements are performed. The generated state is assumed to be reproducible, i.e., not to change substantially over the course of the full measurement. Referring for experimental details elsewhere [10], we note that several characteristics of the system are crucial for obtaining the results described here. The parametric oscillator employed is a monolithic magnesium-oxide-doped lithium niobate resonator which is mechanically stable and located in a tightly temperature-controlled oven. The oscillator's pump wave (frequency 2ω) is obtained by externally resonantly frequency doubling a continuous-wave, diode-pumped,

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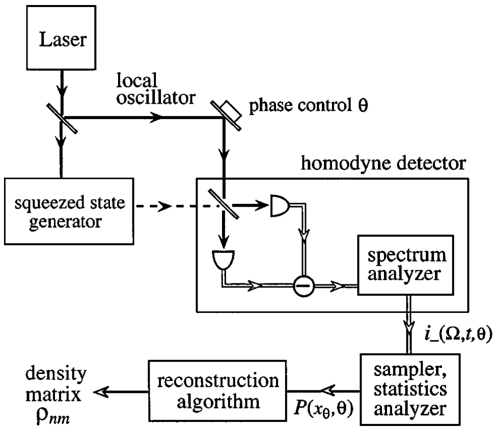

FIG. 1. Block diagram for measurements of the quantum state and the photon statistics. In the squeezed state generator, the second harmonic of the laser frequency generated in an enhancement cavity pumps a monolithic optical parametric amplifier.

miniature Nd:YAG laser ($\lambda = 1064$ nm) of exquisite frequency stability, which also provides the local oscillator wave (frequency ω). This source reliably generates squeezed vacuum with good temporal stability. The squeezed vacuum, whose spectrum is centered at the optical frequency ω, is detected using a balanced homodyne detector. The 2 mW power level local oscillator, whose high-frequency excess intensity noise is suppressed by means of a filter cavity, is overlayed with the unknown state at a 50%-50% beam splitter, and the two resulting waves are detected by highly efficient photodetectors. Since the light fluxes are very high, the detectors do not resolve individual photons but generate a large average current with small fluctuations due to any (quantum and classical) fluctuations of the two waves. An electronic circuit generates the photocurrent difference $i_-(\theta, t)$ which, after accurate balancing of the photodetector electronic gains, is proportional to a particular quadrature of the signal electric field, and to a high degree free of the fluctuations of the local oscillator. Electronic noise is negligible (14 dB below shot noise). The relative phase θ between signal and local oscillator fields is variable, so that all quadratures can be sequentially accessed.

Rather than measuring $i_-(\theta, t)$ directly, for technical reasons we measure phase sensitively its spectral component $i_-(\theta, \Omega, t)$ in a small band around a radiofrequency $\Omega = 2$ MHz by mixing with a rf local oscillator of frequency Ω. In this case, the quantum measurement performed by the homodyne detector is on the two-mode quadrature observable $X(\theta) = e^{i\phi}[a(\omega + \Omega)e^{-i\theta} + a^\dagger(\omega - \Omega)e^{i\theta}] + \text{H.c.}$, where $a(\omega'), a(\omega')^\dagger$ are the annihilation and creation operators of mode ω' [7] and ϕ is the rf local oscillator phase. Thus, the reconstruction performed below yields the quantum state information of the signal's spectral components at $\omega \pm \Omega$.

The data shown in Fig. 2(a) consist of a time trace of $i_-(\theta, \Omega, t)$, taken while the local oscillator phase is scanned. The scan rate is 16 ms$/2\pi$. $i_-(\theta, \Omega, t)$ is obtained by filtering i_- with an rf spectrum analyzer (100 kHz resolution bandwidth, zero span) and sampling its intermediate frequency output every 100 ns with a Nicolet 400 digital oscilloscope. Each sampling corresponds to one measurement of the field quadrature. The statistical distribution of the current i_- for fixed phase θ is the probability distribution $P(x_\theta, \theta)$ of finding a value x_θ for the observable $X(\theta)$. The relation between these quadrature distributions and the density matrix ρ is given by $P(x_\theta, \theta) = \langle x_\theta|\rho|x_\theta\rangle$. By dividing the $\theta = 0, \ldots, 2\pi$ data into 128 intervals, in each of which the phase θ is nearly constant, a set of 128 amplitude probability distributions $P(x_\theta, \theta)$ is obtained. The total number of data points is 160 000. The quadrature amplitudes are normalized by comparison with the distribution measured for a vacuum state (signal blocked) [10].

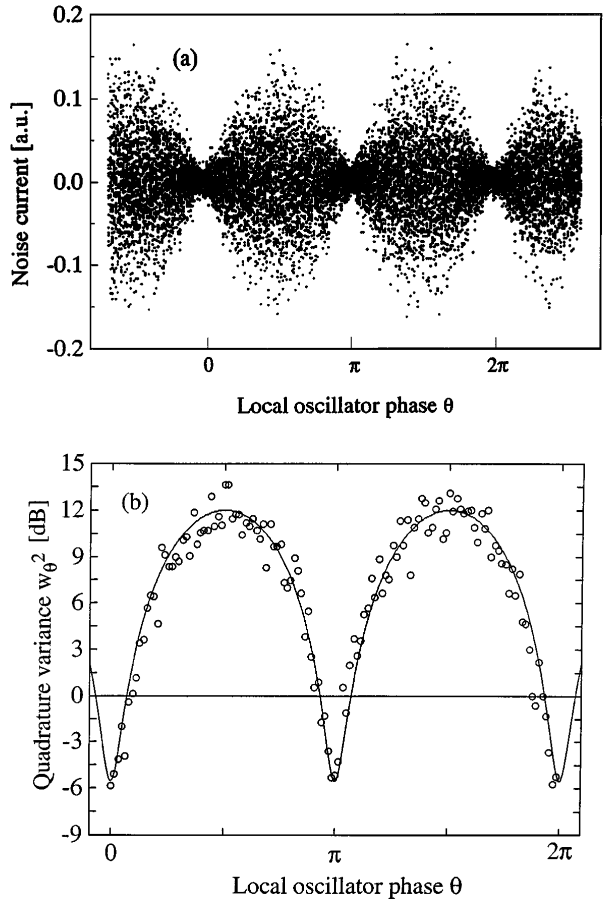

FIG. 2. (a) Record of the photocurrent difference i_- in a 100 kHz bandwidth around 2 MHz as a function of time. The local oscillator phase is varied linearly in time. (b) Variances of the quadrature distributions for 128 local oscillator phases, derived from the data in (a). Solid line is theory, Eq. (1).

The experimentally obtained quadrature distributions agree well with the theoretical distributions for a squeezed vacuum state given by [15]

$$P(x_\theta, \theta) = (\pi w_\theta)^{-1/2} \exp(-x_\theta^2/w_\theta^2),$$
$$w_\theta^2 = (1 + \eta S_-)\cos^2\theta + (1 + \eta S_+)\sin^2\theta, \quad (1)$$

where $1 + \eta S_- = 0.28 \pm 0.03$, $1 + \eta S_+ = 15.8 \pm 1$ are, respectively, the measured variances of the maximally squeezed and antisqueezed quadratures relative to the vacuum [11], obtained from a quantum noise spectral density measurement [10]. $\eta = (82 \pm 5)\%$ is the measured total efficiency which comprises the $(88 \pm 4)\%$ OPA cavity escape efficiency [16], 98% propagation efficiency, 98.5% matching of local oscillator and squeezed vacuum spatial modes, and $(97 \pm 2)\%$ photodetector quantum efficiency. In Fig. 2(b) we report the variances of the measured Gaussian probability distributions and compare them with Eq. (1). This plot is equivalent to a direct measurement of the quadrature variances [6,10].

The reconstruction of the quantum state is performed by evaluating integrals over the measured quadrature probability distributions $P(x_\theta, \theta)$, yielding the density matrix elements in the Fock basis,

$$\rho_{nm} = \int_0^\pi \int_{-\infty}^{+\infty} P(x_\theta, \theta) f_{nm}(x_\theta) e^{i(n-m)\theta} dx_\theta \, d\theta \,. \quad (2)$$

The functions f_{nm} are transcendental functions whose definition and numerical algorithm can be found in Ref. [14].

Of particular interest are the diagonal elements ρ_{nn}, which are the occupation probabilities of the Fock state $|n\rangle$. According to Eq. (2), these depend only on the phase-averaged probability distributions $\overline{P}(x) = \int P(x_\theta, \theta)\, d\theta / 2\pi$ [13]. Figure 3 shows the phase-averaged distributions corresponding to the data of Fig. 2, which agree very well with the theoretical distributions calculated using Eq. (1). Performing the integrations over the averaged quadrature distribution $\overline{P}(x)$, the photon number distribution $p(n) = \rho_{nn}$ shown in Fig. 4 is obtained. For the squeezed vacuum state the probabilities are clearly larger for even photon numbers than for odd ones. For an ideal (minimum uncertainty) squeezed state zero probabilities for odd n are expected, since the Hamiltonian describing the parametric process occurring inside the nonlinear crystal is quadratic in the creation and annihilation operators [17–19]. However, the probabilities for odd photon numbers are nonzero because the squeezed state detected here is a mixed state having undergone losses inside the resonator and during the detection process which cause the distribution to smear out. The photon number distribution for a mixed squeezed vacuum state is given by [18]

$$\rho_{nn} = 2\sqrt{\frac{(2 + \eta S_-)(1 + S_-)}{2 + 2S_- \eta S_-}}\,(\eta s)^n P_n^+((1 - \eta)s)\,,$$
$$\text{with } s = |S_-| / \sqrt{4 + 4S_- + 2\eta S_-^2 - \eta^2 S_-^2}\,. \quad (3)$$

Here P_n^+ denotes the nth Legendre polynomial with positive coefficients, which in terms of the usual Legendre polynomial P_n can be written as $P_n^+(t) = (-i)^n P_n(it)$. As can be seen from Fig. 4, the occupation probabilities calculated from the measured total efficiency η and squeezed quadrature variance $1 + \eta S_-$ agree well with the experimental data.

The photon number average and variance for the experimental photon statistics up to $n = 12$ are $\langle n \rangle_{\text{exp}} = 2.1$ and $\langle (\Delta n)^2 \rangle_{\text{exp}} = 8.5$, respectively. This is in good agreement with the truncated theoretical values 2.17 and 8.86 and shows that the squeezed vacuum state is strongly super-Poissonian. Note, however, that Fock states with $n > 12$, while not accurately reconstructed from the data, do contribute significantly to the average and the variance: the theoretical values including all n are $\langle n \rangle = \eta(S_- + S_+)/4 = 3.5$ and $\langle (\Delta n)^2 \rangle = \langle n \rangle (1 + \eta + 2\langle n \rangle) = 30.1$.

The average photon number has a simple interpretation: it corresponds to the photon flux per unit frequency

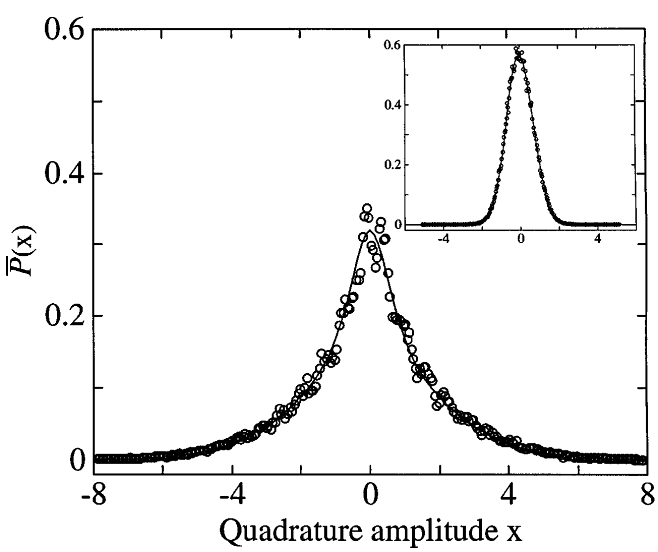

FIG. 3. Phase-averaged marginal distributions $\overline{P}(x)$ of a 5.5 dB squeezed vacuum state, and, in the inset, of the vacuum state. Circles, experimental data; lines, theory. $\overline{P}(x)$ is Gaussian for the vacuum, but non-Gaussian for the squeezed vacuum state. The photon number distribution is obtained by integrations over $\overline{P}(x)$.

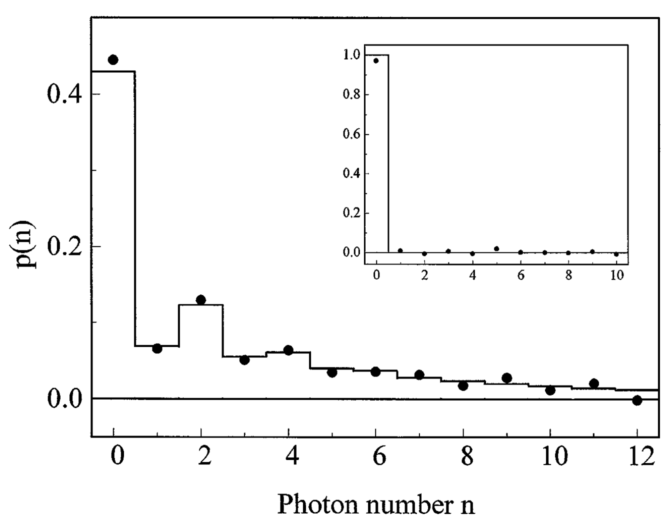

FIG. 4. Photon number distribution of the squeezed vacuum state and the vacuum state (inset). Solid points refer to experimental data, histograms to theory, Eq. (3). The experimentally determined statistical error is 0.03.

interval emitted by the squeezed vacuum source. Integrating over the whole spectrum of the parametric oscillator a total flux of $2\eta\Gamma\pi P_p/(P_{\rm th} - P_p) = 2.6 \times 10^8$ photons per second is calculated for the present device with OPA cavity FWHM linewidth $2\Gamma = 34$ MHz and pump power $P_p = 0.75P_{\rm th}$ [20]. This 40 pW power should be accessible to direct measurement.

The reconstructed density matrix up to $n = 6$ is given explicitly by

$$\rho_{nm} = \begin{pmatrix} 0.44 & 0.00 & -0.23 & 0.00 & 0.14 & 0.00 & -0.08 \\ 0.00 & 0.07 & 0.00 & -0.06 & 0.00 & 0.05 & 0.00 \\ -0.23 & 0.00 & 0.13 & 0.00 & -0.09 & 0.00 & 0.06 \\ 0.00 & -0.06 & 0.00 & 0.05 & -0.01 & -0.04 & 0.00 \\ 0.14 & 0.00 & -0.09 & -0.01 & 0.06 & 0.00 & -0.05 \\ 0.00 & 0.05 & 0.00 & -0.04 & 0.00 & 0.03 & 0.00 \\ -0.08 & 0.00 & 0.06 & 0.00 & -0.05 & 0.00 & 0.04 \end{pmatrix}.$$

The elements are real because the phase $\theta = 0$ was arbitrarily set where the maximally squeezed quadrature occurs in the data of Fig. 2(a). Two features of the density matrix are apparent. The odd diagonals are zero due to the mirror symmetry of the squeezed state Wigner function [10]. Second, oscillations in the magnitude of the elements also occur along the even diagonals.

Evaluating the trace of the square of the density matrix yields a measure of the deviation of the state from a pure one. Using the reconstructed matrix (up to $n = 12$) we obtain $\mathrm{Tr}(\hat{\rho}^2) = 0.48$, the expected value being $1/\sqrt{(1 + \eta S_-)(1 + \eta S_+)} = 0.47$. The mixture character is therefore substantial for our quantum state.

In conclusion, we have reported a complete experimental reconstruction of a particular spectral component of a highly nonclassical state of the light field which exhibits a nontrivial photon number distribution. The even-odd oscillations reflect the quantum (pair) character of the parametric interaction occurring in the nonlinear crystal. This reconstruction shows the power and practical applicability of optical homodyne measurements in combination with recently developed tomographical techniques to pass to the macroscopic world details about a fragile quantum state that carries a very small photon flux. The work presented here may be extended on the one hand to simultaneous analysis of many spectral components covering the full bandwidth of the emitted radiation. On the other hand, the characterization of squeezed light states with small but nonvanishing mean electric field should be rewarding. Here, for sufficiently high degrees of squeezing, direct evidence of quantum interference effects in phase space is expected, resulting in large-period oscillations in the photon number distribution [19].

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